## Planning Algorithms, Steven M. LaValle, Cambridge U. Press Errata in First Printing (May 2006)

August 14, 2020

- Mathematical notation in all captions should be the standard math font used in the main text (rather than the minor variant that appears).
- Page 3: "modeling uncertainties"  $\rightarrow$  "modeling errors"
- Page 25: In the last sentence of the second paragraph of Example 2.1, "inputs" should be "actions".
- Page 28: In the first paragraph, the first sentence under "Alive:" should be "States that have been encountered, but possibly have unvisited next states."
- Page 33: Line 16 should have "iterative deepening" instead of "iterative depending".
- Page 35: Line 6 of the template should be "if x already visited from  $x_G$ " and Line 17 should be "if x' already visited from  $x_I$ "
- Page 39: In (2.10), the last term should be  $l_F(x_F)$ , not  $l(x_F)$ .
- Page 41: In (2.15),  $C_K^*(x_F)$  should be  $C_F^*(x_F)$ . The subsequent sentence should be "This equation looks the same as (2.5) after substituting k = 1; however,  $l_I$  is used here instead of  $l_F$ ."
- Page 42: In (2.16),  $u^{-1}$  should be  $u_k^{-1}$ .
- Page 49: On the first line, delete the word "use" after "logic".
- Page 82: The third sentence after (3.46) should be "The numerator sign selects whether the direction will be above or below the x-axis, and the denominator selects whether the direction will be to the left or right of the y-axis."
- Page 82: Some terms are inverted. In (3.44),  $r_{11}/r_{21}$  should be  $r_{21}/r_{11}$ . Likewise, (3.45) should have  $-r_{31}/\sqrt{r_{32}^2 + r_{33}^2}$  inside of the  $\tan^{-1}$ . These changes cause the arguments in atan2 to be swapped in (3.47) and (3.48). The sentence following (3.49) then becomes, "Note that this method assumes  $r_{11} \neq 0$  and  $r_{33} \neq 0$ ."
- Page 96: In Figure 3.25, the lowest labeled point should have coordinates (2, -1) instead of (0, -1).
- Page 101: In the second paragraph of Further Reading, "it not" should be "is not".
- Page 105: In the 3rd line of Section 4.1, delete "brief".
- Page 106: In the 10th line, "a countable number of open sets" should be "any number of open sets".
- Page 107: The second sentence after the bullets should be "For example, 0 and 1 are boundary points of intervals, (0,1), [0,1], [0,1), and (0,1]."

- Page 109: In the last line of Example 4.5,  $x \mapsto \tan^{-1}(\pi x/2)$  should be  $x \mapsto 2 \tan^{-1}(x)/\pi$ .
- Page 111: The word "open" should be deleted from the caption of Figure 4.4.
- Page 125: The expression at the end of the first paragraph should be  $a_1v_2 + a_2v_1 + v_1 \times v_2$  (rather than a minus sign in front of the cross product).
- Page 126: In the 6th line after "Using quaternion multiplication", insert " with h and -h identified" after "group of unit quaternions".
- Page 132: The line after (4.38) should contain "for which h(x) > 0 if  $x \in \text{int}(\mathcal{C}_{obs})$ " instead of "for which h(x) = 1 if  $x \in \mathcal{C}_{obs}$ ".
- Page 134: After Whitney's embedding theorem is mentioned, the equation is  $m \ge 2n + 1$  not  $m \le 2n + 1$ .
- Page 151: In Problem 19(b), the radius should be 1/2. In Problem 19(c), the radius should be 2/3.
- Page 154: The lower right box of Figure 5.1 should say "C-Space Sampling", not "C-Space Planning".
- Page 155: "subspace"  $Y \to$  "subspace"  $Y \subset X$ , and "domain of  $\rho$  to Y" should be "domain of  $\rho$  to  $Y \times Y$ ".
- Page 167: Before (5.19), it should say a finite set P, rather than simply a set P.
- Page 168: In the 6th line, k = 96 should be k = 196.
- Page 168: In (5.20), N should be k.
- Page 179: Footnote 8 should refer to Section 8.3, not 15.4.
- Page 191: The caption of Figure 5.19 should should end with "gets arbitrarily close to any point in the space".
- Page 193: Caption of Figure 5.22 should be "closest" points, not "close" points.
- Page 194: The sentence "The Kd-tree can be constructed in  $O(n \lg k)$  time." should be "A Kd-tree of k points can be constructed in  $O(nk \lg k)$  time."
- Page 197: Under "Generic preprocessing phase", the q's that appear in the second and third sentences should each be  $\alpha(i)$ .
- Page 204: After Exercise 20, the heading **Implementations** should appear.
- Page 204: In Exercise 5, there should be a  $\frac{1}{\mu(X)}$  in front of the integral.
- Page 204: In Exercise 6, insert "(as a function of i)" after "dispersion".
- Page 208: In the last sentence before Section 6.2.1, delete "that".
- Page 210: In the second line, "refers refer" should be "refers".

- Page 212: Figure 6.4 has been updated to fix an incorrect edge in the roadmap.
- Page 218: In the last sentence of Section 6.2.4,  $l_f$  should be  $f_l$  in two places.
- Page 219: In (6.3), the  $\alpha_i$  must also sum to 1. Thus,

$$[p_1, \dots, p_{k+1}] = \left\{ \sum_{i=1}^{k+1} \alpha_i p_i \in \mathbb{R}^n \mid \alpha_i \ge 0 \text{ for all } i \text{ and } \sum_{i=1}^{k+1} \alpha_i = 1 \right\},$$

- Page 222: The caption of Figure 6.16 should have  $C_{free}$ , not  $C_{obs}$ .
- Page 234: In (6.13), the boundary case of z=1 is true. This results in the slight change to

$$\Phi(z) = \begin{cases} \text{TRUE} & \text{if } z \ge 1\\ \text{FALSE} & \text{if } z < 1. \end{cases}$$

- Page 238: In the last sentence of the first paragraph, "has multiple roots" is changed to "has a double root".
- Page 239: In (6.22), the second = should be  $\geq$ .
- Page 239: In the second-to-last line, "performed" should be "formed".
- Page 247: In the first paragraph of Section 6.5, "motion algorithm" should be "motion planning algorithm".
- Page 248: Six lines from the bottom, NP should be brought out of math mode.
- Page 252: The second sentence of Example 6.9 should be "Since segments in general position may appear multiple times along the lower envelope, the total number of edges is  $\Theta(\lambda_3(n)) = \Theta(n\alpha(n))$ , which is higher than one would obtain from infinite lines."
- Page 255: Exercise 5 should be "shortest-path", not "short-path".
- Page 305: The last line should have "single-destination shortest-path" instead of "single-source shortest-paths".
- Page 309: Should be "the cost is the same in every direction: l(x, u) = l(x, u') for all  $u, u' \in U(x) \setminus \{u_T\}$ ."
- Page 309: The third condition for a navigation function should be "For every reachable state", instead of "For every state".
- Page 322: In the tenth line from the bottom,  $(\phi, U)$  should be  $(U, \phi)$ , and  $(\psi, V)$  should be  $(V, \psi)$ .
- Page 323: In the fourth line from the bottom,  $(V, \phi)$  should be  $(V, \psi)$ .
- Page 338: The fourth sentence (negative definite case) after (8.48) should end with "achieves a local maximum at x."

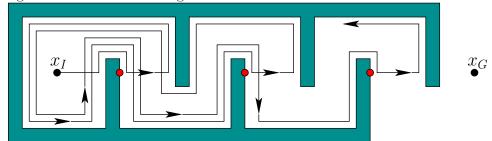
- Page 349: In the fourth line from the bottom, the figure reference should be to Figure 8.13d, rather than Figure 8.12.
- Page 350: In the fourth line, "actions to not" should be "actions do not".
- Page 350: In the ninth line from the bottom, "cubes who vertices" should be "cubes whose vertices".
- Page 359: Caption of Figure 7.41, should be "action sets"
- Page 361: In (9.2), it should be  $u \in (-1,1)$  under the inf.
- Page 367: In (9.13), w should be  $w_i$ , and the following text is added after the equation: "in which  $w_i$  is the component of w corresponding to the particular  $u \in U$ ."
- Page 371: The last sentence of Example 9.10 should be "In this case,  $u^* = u_2$ , which yields the optimal expected regret, calculated as 1 using (9.22)".
- Page 372: In the 4th component of Formulation 9.5,  $Y(\theta) \subset Y$  should be  $Y(\theta) \subseteq Y$ .
- Page 385: Equation (9.64) should be  $(3)w_1 + (-1)(1 w_1) = 4w_1 1$ .
- Page 385: Equation (9.65) should be  $(0)w_1 + (1)(1 w_1) = 1 w_1$ .
- Page 387: In Equation (9.66), L should be  $L_1$ , and Equation (9.67), L should be  $L_2$ .
- Page 406: In Exercise 14, insert "for three players" after "Scissors".
- Page 409: Equation (10.3) should be preceded with  $\theta_k \in \Theta(x_k, u_k)$  and the equation itself should have  $\theta_k$  appearing twice, instead of  $\theta_{k+1}$ .
- Page 410: In Equation (10.5),  $\theta_1$  should be  $\theta_k$ .
- Page 411: In the first sentence after (10.9), "finite value of" should be "finite value if".
- Page 421: In (10.45),  $\theta \in \Theta(x_k, u_k)$  should be  $\theta_k \in \Theta(x_k, u_k)$ .
- Page 423: In Equation (10.47), the denominator is  $2^i$  and the sum equals 7. The following sentence should be "The infinite sum is the standard geometric series and converges to 1" and ends in "converges to 7".
- Page 425: After (10.53), the text should say "in which x' is the next state" instead of "in which  $x' = f(x, \pi(x))$ ".
- Page 429: In Step 4 of the algorithm, it should be "G(x)" that is computed, not " $G(x_s)$ ".
- Page 433: In the 19th line, "last three terms" should be "last four terms".
- Page 433: In (10.72), the last term is  $G_{k+1}^*(x_{k+1})$ , instead of  $G_{k-1}^*(x_{k+1})$ .
- Page 440: The last term of (10.94) should be  $G_{\pi}(x_k)$  instead of  $G_{\pi}(x_{k+1})$ .
- Page 450: The end of "Defining a plan for each player" should have  $z \in Z(x)$ , instead of  $w \in Z(x)$ .

- Page 455: Line 6 of Section 10.6 should have "possible to extend" instead of "possible extend".
- Page 455-456: The sections numbered 10.6.0.1 and 10.6.0.2 should be 10.6.1 and 10.6.2.
- Page 456: In Formulation 10.5, the third item should contain "produces a mode" rather than "produces a state". The fourth item should have  $f((q, m), u, \theta)$  rather than f(q, m), u).
- Page 462: "map of its environment" → "map of the robot's environment"
- Page 465: In the sentence after (11.6), "form" should be "forms".
- Page 469: In (11.20),  $\mathcal{I}$  should be  $\mathcal{I}_{hist}$ .
- Page 472: In Figure 11.3, every "k" should be " $\kappa$ ".
- Page 479: In the last sentence of the first paragraph of Example 11.14, substitute  $p_0$ ,  $p_1$ , and  $p_2$ , for  $p_1$ ,  $p_2$ , and  $p_3$ , respectively.
- Page 480: In the second sentence under "Sensor Feedback", the equation  $\kappa(\eta_k) = y_k$  should be  $\kappa_{sf}(\eta_k) = y_k$ .
- Page 480: Four lines from the bottom, insert "unless 0 is observed" after "will always be the case that  $X_k(\eta_k) = \{1, 2, \ldots\}$ ".
- Page 495: In the second line, y = h(x) should be y = x.
- Page 502: In the bottom of Page 502, "leftmost" should be "rightmost", two times in Example 11.21.
- Page 508: In the second paragraph of Section 11.6.1, the third sentence should start "For example, if  $X = \mathbb{R}^2$  ...".
- Page 505: Caption of Figure 11.27 should have  $H(y_3)$  instead of  $H(y_3x)$ .
- Page 509: The equation,  $\mu_1 = L_1 y_1$ , right above (11.78), should be  $\mu_1 = \mu_0 + L_1 (y_1 C_1 \mu_0)$ .
- Page 509: Equation (11.80) should be

$$\mu_{k+1} = A_k \mu_k + B_k u_k + L_{k+1} (y_{k+1} - C_{k+1} (A_k \mu_k + B_k u_k)).$$

• Page 552: In the first paragraph after "a small modification is needed." the remainder of the paragraph is changed to "The robot remembers the distance to the goal from the last point at which it departed from the boundary, and only departs from the boundary again if the candidate point that is closer to the goal. This is applied iteratively until the goal is reached or it is deemed to be impossible."

• Page 553: Figure 12.23 has been changed to:



- Page 555: There are several small errors (due to the neglect of chattering at the origin) in the paragraph that starts "A nice illustration ...". 4th sentence: Delete the first "directly". 5th sentence: "no more than" should be "exactly". 6th sentence: "all" should be "both". 7th sentence should be "What if the cow is told only that the gate is at least distance 1 away?".
- Page 591: For the 6th example  $(a\dot{x} + b\dot{y} + c = 0)$ , the text should read: "This constraint is similar to the previous one, however the behavior is quite different because the integral curves do not coincide. An entire half plane is reached. It also impossible to stop becasue  $\dot{x} = \dot{y} = 0$  violates the constraint."
- Page 595: Multiply the right side of the last equation in each of (13.8) and (13.10) by -1.
- Page 603: In equation (13.19), the equation for  $\dot{\theta}_1$  should have  $\theta_0 \theta_1$  inside of the sin function.
- Page 609: Some indices are shifted. In the first paragraph, 8th line,  $\dot{q}_i$  should be  $\dot{q}_{i-n}$ . In the second paragraph, the  $\dot{q}$  in the 4th line should be  $\dot{q}_i$ , and in the 8th line,  $h_i$  should be  $h_{i-n}$ .
- Page 612: The first line of Section 13.2.4 should say "unrealistic" instead of "realistic".
- Page 619: In equation (13.59),  $u_2$  should be  $u_u$ .
- Pages 638-639: In (13.156) to (13.160),  $l_1$  and  $l_2$  should be  $\ell_1$  and  $\ell_2$ , respectively.
- Page 642: In the second line of Section 13.4.3.2, "formation" should be "formulation".
- Page 646: Equation 13.201 should be  $\dot{x} = u + \theta$ .
- Page 648: The last equation in (13.207) should have the s variable deleted.
- Page 648: The last paragraph should have "many be more difficult" changed to "may be more difficult".
- Page 653: The second-to-last sentence (regarding  $u_T$ ) of item 5 of Formulation 14.1 should be moved to become the last sentence of item 6.
- Page 695: Equation 14.28 should be

$$\pi(x) = \underset{u \in U_d}{\operatorname{argmin}} \Big\{ l_d(x, u) + G^*(f_d(x, u)) \Big\},\,$$

• Page 695: In the middle of the page, "In that same way" should be "In the same way".

- Page 724: In (15.27), the \* should be a superscript of u.
- Page 725: In (15.30)-(15.35) some i and j subscripts are swapped. In the last term of (15.30), it should be  $f_j$  and  $x_i$ . In (15.31),  $dx_i$  should be  $dx_j$ . In (15.32),  $x_i$  and  $\dot{x}_i$  should be  $x_j$  and  $\dot{x}_j$ . In (15.33),  $\dot{\lambda}$  should be  $\dot{\lambda}_i$ . In (15.34) and (15.35),  $f_i$  should be  $f_j$ .
- Page 740: The last two sentences of the paragraph after Example 13.5 should be "Note that the direction is not fixed because bu is added to all components of  $\dot{x}$ . In general, the set of allowable velocities at a point  $x \in \mathbb{R}^n$  is an m-dimensional hyperplane the tangent space  $T_x(\mathbb{R}^n)$  (if B is nonsingular)."
- Page 740: The second sentence in the second paragraph of Section 13.2.3 should be "Let X be a smooth n-dimensional manifold, and let  $U = \mathbb{R}^m$  for some  $m \leq n$ ."
- Page 741: The second line from the bottom should start "It is sometimes ferred to" instead of "It some times referred to".
- Page 745: In (15.77), the term  $\Delta t g$  should be  $\Delta t (f+g)$ .
- Page 751: In the second row of Example 15.18,  $[h_3, h_1]$  should be  $[h_1, h_3]$ .
- Page 780: Reference [306] should include coauthor S. Har-Peled.
- Page 798: Reference [709] should have Hamilton-Jacobi capitalized.
- Page 785: Reference [413] (G. Sanchez) is out of order.
- Page 803: Reference [854] (J. T. Schwartz and M. Sharir) should be from *Advances in Applied Mathematics*, 12: 298-351, 1983.
- Page 805: Reference [879] (Z. Shiller and S. Dubowski) should have "On computing global time-optimal ..." for the title and the pages are 785-797.