Story Validation and Approximate Path Inference with a Sparse Network of Heterogeneous Sensors

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***Abstract*— Given a story from an agent (sensor outputs from a robot or a tale told by a human) and recordings from a spare network of heterogeneous sensors, this paper provides efficient algorithms that validate whether it is possible to reconstruct a path compatible with the sensor recordings that is also "close" to the agent's story. In solving the proposed problems, we show that effective exploitation of a unique finite automaton structure yields time complexity linear in both the length of the story and the length of the sensor observation history. Besides immediate applicability towards security and forensics problems, the idea of behavior validation using external sensors also appears promising in complementing design time model verification.**

I. Introduction

Surveillance cameras at supermarkets. Occupancy sensors in office rooms. Buried pressure based vehicle sensors at traffic lights. Your thermostat at home. Sensors are everywhere: Due to diminished cost, an ever increasing number of heterogeneous, more reliable sensors are being installed and many are also networked. Some of these sensors are part of smart systems, such as automatic traffic citation issuing systems using cameras installed at cross sections. However, this vast network of sensors is far from realizing its full potential since the integration of sensor data usually requires human intervention. For example, video surveillance systems often merely fuse images from different cameras for human interpretation, which can be error prone. More effective use of networked sensors then calls for the development of specialized and efficient algorithms for these systems.

With the research presented here, we hope to bring us one step closer to the goal of maximizing the utility of networked sensor systems. In [33], we introduced and solved a problem in which an agent's account (a *story*) of its path in an environment is validated against recordings from a sparse network of sensors deployed in the same environment. In that work, an agent's story is required to be error-free and has start/end time matching those of the sensor recordings'. Such formulations are restrictive for two reasons: 1. Even a truthful agent is likely to introduce errors in recalling a long story, especially when the agent in question is a human; 2. Requiring matching start/end time can be hard to guarantee. Moreover, when an agent's story is not consistent with an observation history, the algorithms from [33] do not provide an alternative path for the agent that is "close" to the agent's story. In this paper, we provide highly efficient algorithmic solutions to address and remove all these limitations.

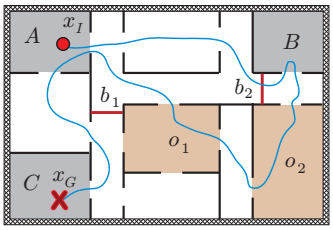

Fig. 1. A workspace with rooms A, B, C, occupancy sensors o_1, o_2 and beam detectors b_1, b_2. The curve connecting the start (circle) and goal (cross) locations is a possible agent path for the story A, B, A, C, which triggers the sensor recordings b_2, o_2, o_1, in that order.

Our study also complements the verification of robotic system design. From the perspective of modeling, a robotic agent in our problem can be viewed as a hybrid system: It goes from room to room to carry out tasks, changing its operation modes along the way. Since the verification of an autonomous system with continuous state space and control input is undecidable in general [2], it is desirable to have external measures to keep in check the unverified portion of such systems. Even for hybrid systems with provably correct designs, such as autonomous robots or cars [8], [15] based on specifying high level tasks using General Reactivity(1) formulas [22], a malfunction could occur due to sensor/actuator/computer errors. Furthermore, complex hybrid systems are also subject to deliberate tempering to behave in unwanted manners. Therefore, automatically monitoring hybrid systems with external sensors can be an effective error correcting and safeguarding measure.

Sensor networks based approaches have been applied to infer basic properties of moving bodies in its range. Networks of binary proximity sensors have been employed to track one or multiple moving bodies using various analytical tools [3], [14], [25], [26]. The task of counting multiple targets is also studied under different assumptions [4], [13]. In these works, the sensors' aggregate sensing range must cover the target(s) at all times, which we do not assume. When only subsets of an environment are guarded, *word problems in groups* [9], [12] naturally arise. For the setup in which targets moving inside a 2D region are monitored with a

set of detection beams, [28] characterizes possible target locations, target path reconstruction up to homotopy, and path winding numbers. In this domain, the surfacing of more interesting behaviors also induces an increase in complexity; few efficient algorithms exist.

In obtaining solutions to more general problems, it becomes clear that the story validation problem we raise can be transformed to the *string edit distance problem between a string and a regular language*, with first algorithmic solution appearing in [30]. Improvements on time and space requirements for algorithms solving such problems can be found in [31], [24], [19], [1]. The best general algorithm for obtaining a string with smallest edit distance from a string x between x and an automaton A appears to be the one given in [1], which takes time $O(|x||A|\lg|A|_Q)$ ($|A|$ is the sum of the number of states and transitions of A and $|A|_Q$ is the number of states of A). For an overview of approximate string matching problems, see [20]. In viewing the resemblance of our problem to the questions asked in [3], [25], [28], our solution follows an *information space* approach [16], which requires the design of a combinatorial filter, similar to those in [17], [29], [32]. These combinatorial filters are minimalist counterparts to widely known Bayesian filters [5], [7], [11], [18], [21], [27].

The rest of the paper is organized as follows. Section II provides formulations of the three problems we study in this paper. Section III briefly reviews the algorithmic solution to the base problem and apply it to solve the first of the three problems. Section IV relates our problem to the string edit distance problem. We then combine the gained insight with our special problem structure to provide algorithms for solving the other two problems with further efficiency gains, in Section V. We also discuss additional extensions in this section and conclude with Section VI.

II. Problem Formulation

A. Workspace, Agent Stories, and Observation History

Let the *workspace* $W \subset \mathbb{R}^2$ be a bounded, path connected open set with a polygonal boundary, ∂W. Let one or more point agents move around in W, carrying out unknown tasks. Every agent has a map of W and may move arbitrarily fast along some continuous path $\tau : [t_0\ ,\ t_f] \to W$. We are interested in a particular agent x which can be thought of as a *suspect*. Agent x provides a *story*, which is a sequence of locations it recalls along its path in increasing chronological order,

$$\mathbf{p} = (p_1, p_2, \ldots, p_n), \quad p_i \subset W.$$

Since p_i's can be viewed as letters from an alphabet, $\mathbf{p}$ can be viewed as a string $p_1 \ldots p_n$ as well; we use the string notation mostly in this paper since it is shorter. We assume that the unique elements of $\mathbf{p}$ are each a simply connected region with a polygonal boundary and pairwise disjoint. The set of all unique elements of $\mathbf{p}$ is denoted $\mathcal{C}_p$, which can be thought of as a set of rooms in W. As an example, for the environment from Fig. 1, agent x may simply report that "I went from room A to room B, then came back to room A, and eventually arrived room C, at which point I stopped." To limit the number of questions that can be posed, we require that agent x starts from p_1 and ends in p_n. Unless otherwise stated in a particular problem, it is assumed that agent x recalls in $\mathbf{p}$ locations on its path correctly.

Let a subset of the workspace W be guarded by a set of occupancy sensors and beam detectors. The placement of sensors in W is unknown to the agents. An occupancy sensor o_i is assumed to detect the presence of an agent in a fixed, convex subset $s \subset W$. For example, a room may be monitored by such a sensor (o_1, o_2 in Fig. 1). A data point recorded by o_i has two parts, an *activation*, $r_{oa} = (o_i, t_a)$, and a *deactivation*, $r_{od} = (o_i, t_d)$. Here t_a is the time when the first agent enters an empty s and t_d is the time when the last agent exits s. A beam detector b_i, on the other hand, guards a straight line segment, $\ell \subset W$, between two edges of ∂W (b_1, b_2 in Fig. 1). A data point of such a sensor is recorded as an agent crosses ℓ, which can be represented by a 2-tuple, $r_b = (b_i, t)$. A beam detector is deactivated right after activation. We further assume that when a beam detector is triggered by an agent, the agent must pass from one side of the beam to the other side.

If we gather all sensor recordings (of the types r_{oa}, r_{od}, r_b) during a time interval $[t_0, t_f]$ and order them by time incrementally, an *observation history* is obtained. This sequence is unique since it is reasonable to assume that no two recordings happen at the exact same time. As we do not assume that an agent provides the exact time when it visits a location, the time in the sensor recording is also relative. Therefore, when we compose the observation history of the sensors, we may discard the time element of each sensor recording, keeping only their relative order. A simplified observation history can be written as (each s_i corresponds to the region covered by a sensor):

$$\mathbf{s} = (s_1, s_2, \ldots, s_m), \quad s_j \subset W.$$

Similar to $\mathbf{p}$, we can write $\mathbf{s}$ as a string. Note that m, n are of comparable size since the more places an agent goes, the more sensor recordings it triggers; the relationship between m, n is linear intuitively. Also, for an occupancy sensor, there should always be an even number of appearances of it in $\mathbf{s}$, with the odd numbered appearances being activation and the even numbered appearances being deactivation. If there is a single agent in the workspace, an activation must also be followed by a deactivation.

In the example given in Fig. 1, a sensor recording of an occupancy sensor could imply that the agent enters and exits from any of the doorways; there are four doorways for o_1 and two for o_2. Similarly, when the beam detector b_2 is triggered, an agent could be passing it from left to right or in the other direction. These sensors certainly cannot distinguish among different agents. Since we work with these weak sensors, the algorithms in this paper apply to a wider range of sensors, provided that they are at least as powerful. We denote the unique sensor regions from $\mathbf{s}$ as $\mathcal{C}_s$. As justified in [33], we assume that any two elements of $\mathcal{C}_p \cup \mathcal{C}_s$ are disjoint (in W).

B. Validation Problems

Given the above setup, we want to know whether a story is consistent with a sensor observation history. For example, if an agent starts from A in the workspace from Fig. 1, it cannot end up at B without triggering any sensor recordings. Our earlier work addressed the problem of validating a single agent's story against the observation history happening during the same time interval, in the presence of no other agent or multiple (an unknown number of) agents in the same workspace. We also assume that the agent's story is exact: The suspect agent x cannot have error its description of a story. The problem of verifying an agent's story of a single agent workspace is given below, which is included here for reviewing the baseline algorithm:

Problem 1 (Exact Story, Matching Time Intervals) *Let there be a single agent in a workspace. The agent provides a story* $\mathbf{p}$ *for the time interval* $[t_0, t_f]$*. During the same time interval, the sensors in the workspace produce an observation history,* $\mathbf{s}$*. Validate whether* $\mathbf{p}$ *is consistent with* $\mathbf{s}$*. Extract a feasible path if the story is consistent.*

The formulation of Problem 1 is quite restrictive in at least two ways. First, requiring the story and the observation history to span the exact same time interval $[t_0, t_f]$ may not be always possible. For example, the sensor network may be inactive at some point due to daily operation schedule or system maintenance. We capture and generalize this case with the following problem:

Problem 2 (Exact Story, Mismatching Time Intervals) *Let there be a single agent in a workspace. The agent provides a story* $\mathbf{p}$ *for the time interval* $[t_0, t_f]$*. The sensors in the workspace produce an observation history,* $\mathbf{s}$*, for a time interval* $[t'_0, t'_f]$ *that overlaps with* $[t_0, t_f]$*. Validate whether* $\mathbf{p}$ *is consistent with* $\mathbf{s}$*. Extract a feasible path if the story is consistent.*

The second restriction is that a truthful agent, be it human or robot, may inevitably make mistakes in recalling a story. One very likely scenario here is that the agent may have missed in $\mathbf{p}$ locations in $\mathcal{C}_p$ it has visited. For instance, a truthful agent may have recalled a story $\{A, B, C\}$ although its story is actually $\{A, B, A, C\}$. This scenario translates into the formulation below (For two strings/sequences $\mathbf{p}, \mathbf{p}'$, the expression $\mathbf{p} \leq \mathbf{p}'$ or $\mathbf{p}' \geq \mathbf{p}$ denotes that $\mathbf{p}$ is a subsequence of $\mathbf{p}'$):

Problem 3 (Partial Story, Matching Time Intervals) *Let there be a single agent in a workspace. The agent provides a story* $\mathbf{p}$ *for the time interval* $[t_0, t_f]$*. During the same time interval, the sensors in the workspace produce an observation history,* $\mathbf{s}$*. Let* $\mathbf{p}' \geq \mathbf{p}$ *be a sequence with elements from* $\mathcal{C}_p$*. Validate whether there exists a* $\mathbf{p}'$ *that is consistent with* $\mathbf{s}$*. If such a story* $\mathbf{p}'$ *exists, find one of shortest length.*

Problem 3 motivates a more general case: The agent, even with best effort, may add locations it has not visited (insertion), miss locations it has visited (deletion), or report some locations it has visited incorrectly (substitution). As an example, a truthful agent may have recalled $\{A, B, C\}$ when the actual story is $\{C, B, B, D\}$. Calling each of the three possibilities to introduce an error as an *edit*, we obtain the following problem:

Problem 4 (Error in Story, Matching Time Intervals) *Let there be a single agent in a workspace. The agent provides a story* $\mathbf{p}$ *for the time interval* $[t_0, t_f]$*. During the same time interval, the sensors in the workspace produce an observation history,* $\mathbf{s}$*. Let* $\mathbf{p}'$ *be a sequence with elements from* $\mathcal{C}_p$*. Find a* $\mathbf{p}'$ *that is consistent with* $\mathbf{s}$ *such that* $\mathbf{p}$ *can be obtained from* $\mathbf{p}'$ *with the least number of edits.*

We point out that besides the above problems, many other interesting formulations are possible. For example, one may require the suspect's story have a maximum error rate of 10%, for which a solution can be derived from the solutions of Problem 4. Due to limited space, we focus on the general cases given in Problem 2 through 4; additional extensions are then discussed briefly.

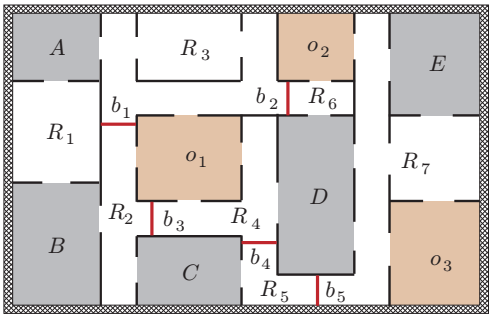

Fig. 2. A workspace with five beam detectors b_1 through b_5, three occupancy sensors o_1 through o_3, and five labeled rooms A through E. Thus, $\mathcal{C}_p = \{A, B, C, D, E\}$, $\mathcal{C}_s = \{b_1, b_2, b_3, b_4, b_5, o_1, o_2, o_3\}$. There are seven connected components R_1 through R_7 when regions guarded by sensors and rooms are treated as workspace obstacles.

III. Baseline Algorithm and the Case of Mismatching Time Intervals

In this section we review the baseline algorithm for Problem 1, with the example setup from Fig. 2. Only the most essential elements from [33] are reproduced here; readers may check that paper for a more thorough explanation. After the recapitulation, we apply the algorithm to solve Problem 2.

A. Baseline Algorithm

It is observed that, when regions covered by sensors and rooms are not occupied, they act as obstacles. In the example, these obstacles partition the workspace into seven connected components, R_1 through R_7. Assuming that the workspace, rooms, and sensors are given to us as edge lists, we apply a cell decomposition procedure [6] to extract these connected

components, which is then transformed into a graph structure that captures the connectivity of the rooms and sensors. We call this graph G the *connectivity graph*, which extracts all necessary information needed for validating an agent's story. The connectivity graph for our example is given in Fig. 3. Each beam detector has two vertices in the graph to represent its two sides; these are marked differently from other vertices. To avoid making the graph too complicated for viewing, some of the edges are collapsed and self loops of vertices from $\mathcal{C}_p$ are not shown (An agent can always go out of a room and come right back without triggering any sensor recordings).

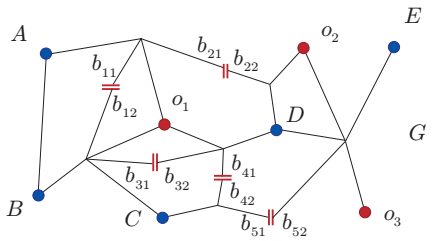

Fig. 3. The connectivity graph G for the example given in Fig. 2.

With the construction of the connectivity graph G, Problem 1 effectively becomes graph search. Note that at any given moment, only part of G is available, depending on where the agent is and which sensor is active. Although Fig. 3 is shown as a single graph, searching through it using breath first or depth first queuing leads to exponential number of branchings with respect to the length of $\mathbf{s}$ due to beam detectors. Therefore, we need to explore additional structure in the problem. For story $\mathbf{p} = p_1 \dots p_n$ and observation history $\mathbf{s} = s_1 \dots s_m$, since the search must march through both strings forward, a position in the two strings and a location in the graph G define a subproblem that is just the same as the initial problem. For example, a subproblem may be validating $p_i \dots p_n$ against $s_j \dots s_m$ starting from room E. There is clearly only a polynomial number of subproblems; if the time it takes to go from one subproblem to a smaller one is also polynomial, then the overall search is efficient in input size.

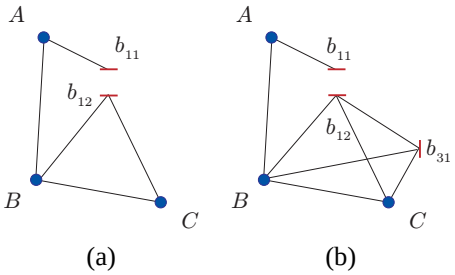

Fig. 4. a) The available subgraph of the example from Fig. 2 when b_1 is the first sensor recording. b) The available subgraph of the example from Fig. 2 for the recording $b_1 b_3$.

It turns out that this is indeed the case for Problem 1. For illustration, suppose that the story is $\mathbf{p} = ABDEC$ and the sensor observation history is $\mathbf{s} = b_1 b_3 o_2 o_2 b_4$. First, we construct a graph based on G and $\mathbf{s}$ that captures all possible agent paths for a fixed $\mathbf{s}$. Since agent x starts at A and must first pass b_1, before b_1 is triggered, the part of G that is available is G_1 (see Fig. 4(a)). Similarly, the next part of G available, after agent x's passing of b_1 but not b_3, is G_2 (see Fig. 4(b)). For all such G_j's, agent x must pass through them sequentially. Observing this, G_j's are chained together to get a composite graph G_s as illustrated in Fig. 5, in which the directed edges allow only one-way passages.

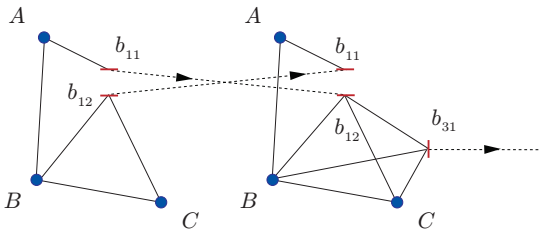

Fig. 5. Part of the composite graph that agent x must pass through.

The rest of the algorithm is dynamic programming much like Dijkstra's algorithm[10]: We recurse over both $\mathbf{p}$ and G_s. More specifically, for each p_i of $\mathbf{p}$, we mark in G_s the possible vertices that agent x can be at after going through $p_1 \dots p_i$. Even though G_s has up to mn_w (Let n_w be the size of input W_{free}) vertices, there are at most $m+1$ vertices corresponding to p_i, one in each $G_j, j = 1, \dots, m+1$. For p_i in G_j (if there is one), we may denote it as (p_i, j). We are left with deciding the total time needed for finding all $\{(p_i, j)\}$ reachable from $\{(p_{i-1}, j)\}$ via 1-hop(Passing a single element of $\mathcal{C}_p$). This is not simply checking neighbors for each (p_{i-1}, j) with fixed i, since passing a sensor vertex is a 0-hop. Here we use the special structure of G_s again. Since (p_i, j) belongs to G_j, it can be reached in only two ways: From the start vertices of G_j or from (p_{i-1}, j). This insight yields again a dynamic programming subroutine, with the cost of $O(m \lg n_w)$ per p_i. The total complexity for searching G_s is then $O(nm \lg n_w)$.

The pseudocode is summarized in Algorithm 1. The subroutine BUILDCONNGRAPH(W_{free}) returns the connectivity graph G given an edge list representation of W_{free}. The subroutine SUBG(G, v_1, v_2) returns the available part of G starting from v_1 and ending at v_2. Finally, CHAIN(...) connects all input graphs sequentially. The time spent on these portion of the algorithm is $O(mn_w \lg n_w + n_w^2)$ [33]. If VALIDATEAGENTSTORY returns true, which means $\mathbf{p}$ is consistent with $\mathbf{s}$, a possible path can be retrieved via backtracking the search process. We now apply Algorithm 1 to Problem 2.

B. Story and Observation History with Mismatching Time Intervals

Problem 2 has several subcases, depending on how $[t_0, t_f]$ and $[t'_0, t'_f]$ compare. Assuming that t_0, t_f, t'_0, t'_f are pairwise

Algorithm 1 VALIDATEAGENTSTORY

Input: W_{free}, $\mathbf{p} = (p_1, \ldots, p_n), \mathbf{s} = (s_1, \ldots, s_m)$
Output: **true** if $\mathbf{p}$ is consistent with $\mathbf{s}$, **false** otherwise

```
G ← BUILDCONNGRAPH(W_free)
for j = 1 to m + 1 do
    G_j ← SUBG(G, j == 1?p_1 : s_{j−1}, j = m + 1?nil : s_j)
end for
G_s ← CHAIN(G_1, ..., G_{m+1})
initialize V_s, V_s' as empty sets of two tuples
V_s ← {(p_1, 1)}    //    A two tuple is a vertex of G_s
for i = 2 to n do
    for j = 1 to m + 1 do
        if (p_i, j) adjacent to (p_{i−1}, k) ∈ V_s for some k ≤ j then
            if i == n&&j == m + 1 then
                return true
            end if
            add (p_i, j) to V_s'
        end if
    end for
    V_s ← V_s'; empty V_s'
end for
return false
```

different, the following six cases are possible:

$$t_0 < t_f < t_0' < t_f', \quad t_0 < t_0' < t_f < t_f',$$
$$t_0' < t_0 < t_f < t_f', \quad t_0 < t_0' < t_f' < t_f,$$
$$t_0' < t_0 < t_f' < t_f, \quad t_0' < t_f' < t_0 < t_f.$$

For the first and last cases, $[t_0, t_f]$ and $[t_0', t_f']$ are disjoint. In these cases, the sensors cannot possibly observe the agent during $[t_0, t_f]$; nothing needs to be done about $\mathbf{p}$ (as long as $\mathbf{p}$ does not conflict with itself). We are not yet done, because we still need to check whether an empty story is consistent with $\mathbf{s}$. This can be done using VALIDATEAGENTSTORY (The search portion only takes $O(m \lg n_w)$ time).

The second case is $t_0 < t_0' < t_f < t_f'$, which means that a later portion of agent story overlaps with an earlier portion of the sensor observation history. This can be handled by running VALIDATEAGENTSTORY algorithm n times. For the i-th run, the story is $(p_i, \ldots, p_n)$ and the sensor observation history is $\mathbf{s}$. If any of the runs returns true, then the story is valid; otherwise the story is inconsistent. We note that the search can be arranged more efficiently by working on the same p_i's of different runs at the same time; thus, the time complexity for this case remains $O(nm \lg n_w)$.

For the third case, $t_0' < t_0 < t_f < t_f'$, the story $\mathbf{p}$ may start in the middle of G_s. VALIDATEAGENTSTORY can be easily modified to handle this: Instead of starting in G_1, we now allow the search to start at (p_1, j) for all applicable j's. If p_n is ever reached in the search, the modified algorithm should report that $\mathbf{p}$ is consistent with $\mathbf{s}$. The time complexity again remains the same. Following along the same lines, we can decide cases four and five.

IV. THE COMPOSITE AUTOMATON STRUCTURE

Although solution to Problem 2 is a relatively straightforward extension of the VALIDATEAGENTSTORY algorithm, it is not immediately clear whether the approach applies to Problem 3, 4, and more general cases. Part of the difficulty comes from the composite graph G_s: It appears to have more structure than a standard directed graph with $O(mn_w)$ vertices. On the other hand, we observe that VALIDATEAGENTSTORY operates much like an automaton in the sense that it processes $\mathbf{p}$ sequentially and either accepts or rejects. This prompts us to ask: Can we turn G_s into an automaton and apply results from Automata Theory to tackle our problem? We answer the first part of this question in this section and delay the second part to the next.

The conversion of a connectivity graph G and a observation history $\mathbf{s}$ into finite automata is relatively straightforward. For each pair of consecutive sensor recordings s_i, s_{i+1} (except when $i = 0, m$), we isolate the part of G that can reach vertices of s_{i+1} from vertices of s_i and convert it to an automaton. When $i = 0$, we let the start state of the automaton transit to all vertices in $\mathcal{C}_p$ and when $i = m$, we let all vertices in $\mathcal{C}_p$ be acceptance states. These automata are then chained together to give a composite automaton.

As a concrete example, we assume again that the observation history is $\mathbf{s} = b_1 b_3 o_2 o_2 b_4$. The first sensor recording is b_1, which means that agent x must at one point reach either b_{11} or b_{12} and cross the beam detector. From Fig. 3 it is clear that if agent x starts from D or E, it is impossible for the agent to get to b_1 without triggering other sensors. Therefore, we only need to consider A, B, C, b_{11}, and b_{12}. Coincidentally, this gives us that part of G available is the same as 4(a). We may convert this to the automaton in Fig. 6(a).

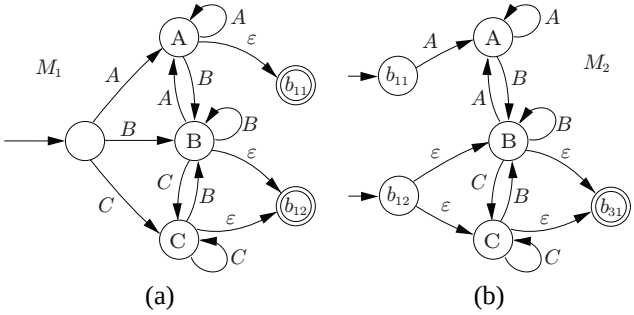

Fig. 6. a) The corresponding NFA when b_1 is the first sensor crossed. b) The NFA for consecutive sensor recordings $b_1 b_3$.

The sensor that is activated next is b_3. Since agent x must start from b_{11} or b_{12}, the connectivity graph G tells us that the agent can only pass b_3 from left to right. The part of G available in this case is the same as 4(b). Follow similar construction, we obtain the corresponding automata shown in Fig. 6(b). Note that Fig. 6(b) represents two automata with one starts at from b_{11} and one starts from b_{12}. The rest of the sensor recordings from $\mathbf{s}$ can be processed in the same manner. For an observation history $\mathbf{s}$ of length m, $m + 1$ automata are obtained. Denote these automata M_1 through M_{m+1}. For implementation purpuse, we connect all of M_{m+1}'s states to a single acceptance state F via an ϵ transition. It is straightforward to observe that a story $\mathbf{p}$ is consistent with $\mathbf{s}$ if and only if the story string can be

partitioned into pieces $P_1, \ldots, P_{m+1}$ such that P_i is accepted by M_j. Alternatively, if we connect M_1 through M_{m+1} together to get a *composite automaton*, M (Fig. 7), then $\mathbf{p}$ must drive M from start state to F.

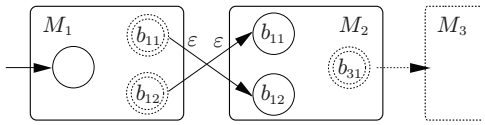

Fig. 7. Sketch of the composite automaton for the observation history string $b_1 b_3 o_2 o_2 b_4$.

With minimal effort, VALIDATEAGENTSTORY can be modified to work with the composite automaton M, which will be able to resolve Problem 1 and 2, keeping the dynamic programming framework intact. Although it does not make the algorithm more efficient, the approach provides an alternative interpretation of these problems: Searching $\mathbf{p}$ through G_s now becomes simulating M over $\mathbf{p}$, making these problems tests of whether a string belongs to a regular language.

V. GENERALIZATIONS AND EXTENSIONS

For Problem 1 and 2, only a few stories are checked against an observation history $\mathbf{s}$. This is no longer the case for Problem 3 and 4 since an infinite number of possible stories must be checked in the process of solving these problems. This is where the automaton structure comes in: If we simulate a story $\mathbf{p}$ over an automaton, we know that on seeing p_i, there are a set of fixed states the automaton can be in. As long as we maintain these finite number of states, an infinite number of string patterns can be handled. In fact, existing results from Automata Theory have completed part of the job for us: Problem 3 and 4 can be viewed as the *string edit distance problem between a string and an automaton* (with the string being $\mathbf{p}$ and the automaton being M), for which efficient algorithms exist.

Our job is not done, however. Exisitng algorithms assume that the string $\mathbf{p}$ and the automaton M are the inputs. In our problem, the automaton M is an intermediate input built from $\mathbf{s}$ and G_s; thus, a direct adoption of approximate string matching algorithms may not fully explore the structure of M. In this section, we explore how the sequential nature of M allows us to subdivide Problem 3 more effectively than off-the-shelf approaches. After solving Problem 3, we sketch at high level how the same structure helps solve Problem 4 as well.

A. Partial Story

We continue to work with the example from Fig. 2 and assume that the story is $\mathbf{p} = ABDEC$ and the observation history is $\mathbf{s} = b_1 b_2 o_2 o_2 b_4$. Suppose that we obtain automata M_j's as well as M from $\mathbf{s}$ already. With the analysis from Section IV, Problem 3 becomes finding a shortest $\mathbf{p}' \geq \mathbf{p}$ such that $\mathbf{p}'$ is accepted by M. The problem may seem a bit daunting at first glance: An infinite number of $\mathbf{p}'$ needs to be examined, as long as $\mathbf{p}' \geq \mathbf{p}$.

However, any consistent $\mathbf{p}'$ must drive the composite automaton M to an accepting state. If we assume that some $\mathbf{p}' \geq \mathbf{p}$ is one shortest such that is accepted by M, then it must have the format $\mathbf{p}' = \omega_1 A \omega_2 B \omega_3 D \omega_4 E \omega_5 C \omega_6$, in which ω_i's denote the parts missing from $\mathbf{p}$. Since $p_1 = A$, we search M and find shortest string from the start state of M to all copies of A in M (We may denote the copy of A from M_j as (A, j)); there are up to $m+1$ of these. Denoting these strings as $\sigma(1, 0, j)$. For each j there may be multiple such strings of the same shortest length; in this case any of these will do. By the principle of dynamic programming, $|\omega_1| + 1 = |\sigma(1, 0, j)|$ for some j. Moving to $p_2 = B$, for each (B, k) let us denote a shortest string taking M from some (A, j) to (B, k) as $\sigma(2, j, k)$. Among these $\sigma(2, j, k)$'s for a fixed k, we need to get one such so that $\sigma(1, 0, j) + \sigma(2, j, k)$ is a shortest. Again, the principle of dynamic programming tells us that for some j, k, $|\omega_1| + 1 = |\sigma(1, 0, j)$ and $|\omega_2| + 1 = \sigma(2, j, k)|$. Following this method, if some simulation branches remain after all of $\mathbf{p}$ are exhausted, then a consistent $\mathbf{p}'$ exists and one can be extracted via backtracking the search process.

Algorithm 2 VALIDATEPARTIALSTORY

Input: $\mathbf{p} = (p_1, \ldots, p_n, F), M, M_1, \ldots, M_{m+1}$
Output: **true** if M accepts some $\mathbf{p}' \geq \mathbf{p}$, **false** otherwise

```
1: initialize 2D array L as array of ∞'s
2: L(0,1) ← 0
3: for i = 1 to n + 1 do
4:    for j = 1 to m + 1 do
5:       ℓ ← ∞
6:       for each start state S_k of M_j do
7:          t ← {min_{j'} SHORTESTLEN((p_{i-1}, j'), S_k)}
8:          ℓ ← min{ℓ, t + SHORTESTLEN(S_k, (p_i, j))}
9:       end for
10:      L(i, j) ← min{ℓ, SHORTESTLEN((p_{i-1}, j), (p_i, j))}
11:   end for
12: end for
13: if L(n + 1, m + 1) ≠ ∞ then
14:    return true
15: end if
16: return false
```

With above analysis, it becomes clear that the insight enabling the reduction of a factor of m in VALIDATEAGENTSTORY also applies here. That is, in finding the shortest string containing $p_1 \ldots p_i$ and taking M from the start state to (p_i, k), we do not need to look at (p_{i-1}, j) for all $j \leq k$. Instead, we only need to know the shortest string that takes $(p_{i-1}, j), j \leq k-1$ for some j, to the start state(s) of M_k; the rest of the search is limited to M_k. Since searching inside M_k for shortest path can be done with Dijkstra's algorithm in $O(n_w^2)$, the overall running time for the search part of validating a partial story is $O(nmn_w^2)$. The pesudocode is described in Algorithm 2. Note that we append to $\mathbf{p}$ an element F, which is the acceptance state of M. Element $L(i, j)$ of the 2D array L stores the length of the shortest string that drives M to the state (p_i, j) and contains $p_1 \ldots p_i$ as a subsequence. The subroutine SHORTESTLEN(a, b) returns the length of

the shortest string that takes M from state a to state b. As discussed, we can obtain SHORTESTLEN$((p_{i-1}, j'), S_k)$ incrementally.

B. Story with Errors

Although Problem 4 appears harder than Problem 3, it is not clear that it is more time consuming. On one hand, to allow insertion, deletion, and substitution of story string $\mathbf{p}$, we need to know the shortest edit distance to reach all states of M for each p_i, instead of knowing only the shortest distance to states (p_i, j). Denote this distance $D(p_i, T)$ in which T is a state of M and let $D(p_i, S, T)$ be the shortest edit distance from state S (of M) to T on p_i, we obtain the recursion

$$D(p_i, T) = \min_S\{D(p_{i-1}, S) + D(p_i, S, T)\}.$$

For a general automaton of Q states, $D(p_i, S, T)$ requires $O(Q^3)$ ($O(m^3 m_w^3)$ for our problem) computation time using a modified all pairs shortest path algorithm [30]. In our case, since $D(p_i, S, T)$ is ∞ for $S \in M_k, T \in M_j$ when $j < k$, we may subdivide the calculation of $D(p_i, T)$ further, staged at each M_j. Suppose T is an internal state of M_j (not start/end states), $\{S_j\}$ are the start states of M_j (there are at most two of these), then the shortest edit distance from $S \in M_{j-1}$ to T, passing some $\{S_j\}$ can be obtained as

$$\min_S\{D(p_{i-1}, S) + \min\{\min_{S_j}\{D(p_i, S, S_j) + D(\epsilon, S_j, T)\},\\ \min_{S_j}\{D(\epsilon, S, S_j) + D(p_i, S_j, T)\}\}\}.$$

We can regroup the formula as

$$\min_{S_j}\{\min\{\min_S\{D(p_{i-1}, S) + D(p_i, S, S_j)\} + D(\epsilon, S_j, T),\\ \min_S\{D(p_{i-1}, S) + D(\epsilon, S, S_j)\} + D(p_i, S_j, T)\}\}.$$

Hence, instead of $O(m^3 m_w^3)$ time, the sequential structure of M enables us to cut the processing time per p_i to at most $O(m m_w^3)$.

On the other hand, unlike in Problem 3, with the introduction of deletion and substitution, a matching story is always possible for Problem 4, as long as the sensor recordings are self-consistent (That is, the language of M is not empty): In the worst case, we can change $\mathbf{p}$ to the shortest string accepted by M via substitution, followed by insertion if $\mathbf{p}$ is too short and followed by deletion if $\mathbf{p}$ is too long. If we denote the length of the shortest string accepted by M as n', then a $\mathbf{p}'$, accepted by M and closest to $\mathbf{p}$, cannot have length more than $\max\{n, n'\}$. This is also the maximum number of edits necessary.

From above observation, we see that it is not necessary to carry all pairs shortest path search over M for each p_i. This observation is the same one that enables the approach from [1] to keep only two levels of a transducer structure in memory. We adapt the same transducer construction to our problem, which has the high level structure illustrated in Fig. 8. Denote this transducer U. Comparing U with that from [1], our transducer has additional structures: It is directed not only from top to bottom but also from left to right. This means that searching U can be partitioned into searching individual M_j^i's from left to right then top to bottom. For Problem 3, finding $\mathbf{p}'$ with smallest number of edits is equivalent to finding accepting string of U with with the smallest cost. This is then a single source shortest path problem on U. As said, for each i, we carry out the search from left to right from M_1^i to M_{m+1}^i, which takes time $O(m \cdot m_w \cdot m_w \lg m_w)$. Doing this for all i (top to bottom) then takes time $O(nmm_w^2 \lg m_w)$. In contrast, preprocessing along takes $O(m^3 n_w^4)$ in [30] (Recall that m, n are of comparable lengths). Our result is also slightly better than the (more general) algorithm presented in [1], which has time complexity $O(nmm_w^2 \lg(mm_w))$ in this context.

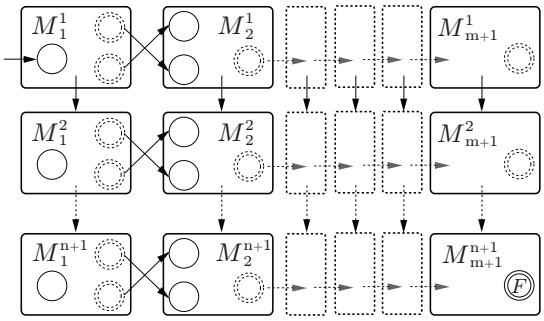

Fig. 8. Sketch of the transducer built from string $\mathbf{p}$ and automaton M.

C. Additional Extensions

Having provided algorithms for Problems 2 through 4, many additional questions can be answered. We briefly discuss some of the possible extensions here.

Constraints. Besides the restrictions placed in Problem 1 through 4, many other problem variations are also possible and can be resolved similarly. The case that requires the suspect's story having a maximum error rate of 10% is one such constraint. As another example, the agent may be confident about certain parts of its story. In Problem 3, this translates to some substrings of the story string must be processed without inserting additional locations. Yet another example is that the story and/or the observation history may contain time "gaps", as an extension of Problem 2. These additional constraints can be dealt with by modifying the above algorithms.

Multiple agents. What if there are multiple agents in the workspace? As analyzed in [33], when an unknown number of agents are in the workspace, not all sensor recordings are triggered by agent x; there are $2^{|\mathbf{s}|}$ possible observation history by agent x. Moreover, the observation history has a different structure: The active occupancy sensors act as hubs that connect parts of the workspace together. Any agent can then pass an active occupancy sensor multiple times. Carrying over the analysis, we observe that the "unit automaton structure" still holds: A composite automaton can be built with individual automata having no more than $O(m_w)$ states. Such orthogonality allows direct combination of the multiple agent case from [33] with the solutions of

Problems 2 through 4, generalizting them to include multiple agent cases.

VI. Conclusion and Future Directions

In this paper, we studied the problem of validating an agent's story, which may be partial or contain errors, against the observation history of external sensors. In addition to deciding the validity of an agent's story in time linear in both the length of the story and the length of the observation history, our algorithms produce a path compatible with the sensor observations that is also closest to the agent's story in terms of string edit distance, whenever such a path exists.

Many intriguing questions remain. Focusing on the discrete setting, the next natural question appears to be the validation of multiple agents' combined story: It is possible that several agents' stories are valid when validated individually but problematic when put together. On the other hand, if we consider differential constraints for the agents, two immediate qualitative implications arise. First, since an agent's speed is limited, the exact time of sensor recordings, otherwise neglected, becomes relevant in filtering out long paths between sensor locations. Second, it becomes possible to measure how far away an agent's behavior deviates from the optimal behavior (In terms of distance traveled, for example). Besides these immediate extensions, another interesting direction is to study optimal sensor placements for the detective task, which was discussed to some extent in [23], [28]. Finally, we have only studied part of the spatial and temporal features of a story. With logic, it may become possible to interpret more challenging agent behavior such as "how" and "why".

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