Agglomerative Clustering on Range Data with a Unified Probabilistic Merging Function and Termination Criterion[1]

Steven M. LaValle Kenneth J. Moroney Seth A. Hutchinson
Dept. of Electrical and Computer Engineering
University of Illinois, Urbana, IL 61801

Abstract: *Clustering methods, which are frequently employed for region-based segmentation, are inherently metric based. A fundamental problem with an estimation-based criterion is that as the amount of information in a region decreases, the parameter estimates become extremely unreliable and incorrect decisions are likely to be made. We show that clustering need not be metric based, and further we use a rigorous region merging probability function that makes use of all information available in the probability densities of a statistical image model. Also, by using this probability function as a termination criterion, we can produce segmentations in which all region merges were performed above some level of confidence.*

1 Introduction

Clustering techniques have been popular for image segmentation. Given an appropriate feature space (or parameter space), a feature value is computed for each image element (either a pixel or a region). Similarity is determined by the metric distance between values in the feature space. In contrast to this, we define "similarity" as a Bayesian, model-based posterior probability that the union of two regions is homogeneous.

Clustering has been applied to a variety of image types and models. For instance, Silverman and Cooper [5] segment intensity images into regions that can be approximated by planar or quadric surfaces. Four basic components involved in most clustering algorithms are [1]: 1. Define a feature metric space. 2. Determine feature values corresponding to pixels or regions. 3. Iteratively group pixels or regions with close features in the metric space 4. Terminate based on some stopping criterion (if the number of classes is unknown). The feature space could, for example, correspond directly to pixel intensities, or could represent a space of polynomial surfaces, as in [5]. The decisions involved in the third step depend on the particular clustering algorithm chosen, such as agglomerative clustering and K-means clustering. Most clustering algorithms require specification of the number of classes, and recent work has been done specifically addressing the problem of determining the number of classes, known as *cluster validation*, in the context of image segmentation applications [2, 6].

At the core of our approach is the determination of the probability that the union of two regions is homogeneous, given an implicit surface model and a noise model, presented in [4]. This probability is used to make region merging decisions, and can also be used as a termination criterion. This work represents a significant contribution to segmentation since:

- Our merging criterion uses all of the information (in a statistical sense) contained in the image model, unlike estimation-based approaches.
- The region merging probability presents a rigorous measure of confidence in the performed merge.
- We have a Monte Carlo based method for directly computing the integral of a conditional density over the parameter space of implicit polynomial surfaces.

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1. Generate a set of small regions, $\mathcal{R}$.
2. Select the pair of adjacent regions, R_i and R_j, that maximizes: $P(H(R_i \cup R_j)|\mathbf{y_i}, \mathbf{y_j})$.
3. Replace R_i and R_j with $R_i \cup R_j$ in $\mathcal{R}$.
4. If $P(H(R_i \cup R_j)|\mathbf{y_i}, \mathbf{y_j}) < P_c$ then terminate
5. Go to 2

Figure 1. Our agglomerative clustering algorithm.

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2 The Algorithm

We are initially presented with a disjoint set of regions, $\mathcal{R}$, which represents a partition of the image into connected sets. These could be obtained as the result of a region-splitting procedure, or by dividing an image into square regions of equal size as in [5]. For our experiments, we obtained an initial region set by combining a plane-fitting, region-splitting procedure with the Canny edge detector.

Figure 1 shows our agglomerative clustering algorithm. We replace the usual metric-based criterion in Step 2 with a criterion based on the region merging probability. For some image subset, R_k, we use the predicate $H(R_k)$ to represent the condition that R_k is homogeneous. The vectors

[1] A complete version of this paper can be obtained by sending email to lavalle@cs.uiuc.edu.

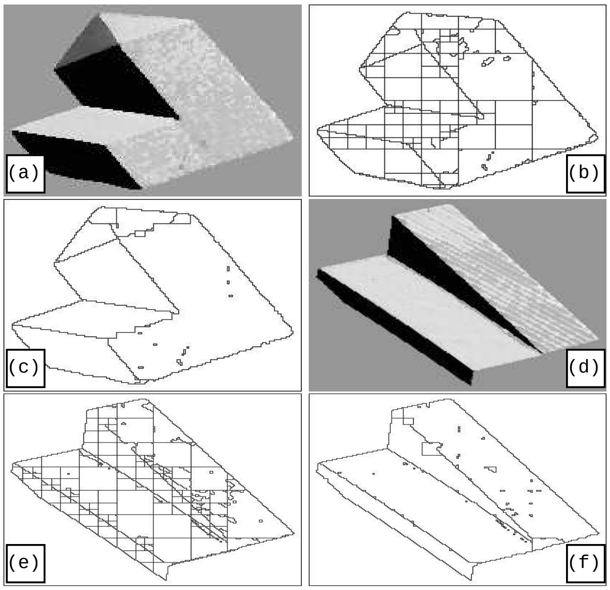

Figure 2.

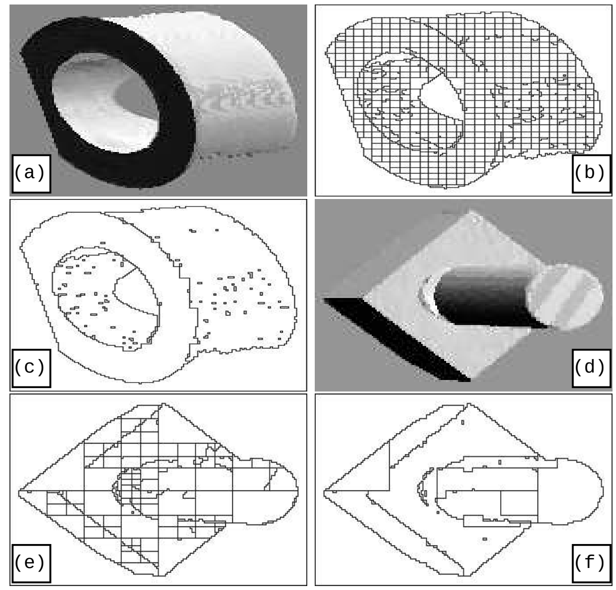

Figure 3.

$\mathbf{y_i}$ and $\mathbf{y_j}$ correspond to model-based information obtained from R_i and R_j, which is described in [4]. We can request a segmentation in which all merges with probability above P_c have been performed.

3 Experiments and Conclusions

For our experiments we developed a Monte Carlo based computation scheme to evaluate high-dimensional integrals that result from the Bayesian region merging probability [3]. Figures 2 and 3 show some results on real range images, using planar and quadric models, respectively. Figures 2.a, 2.d, 3.a, and 3.d show artificial renderings of the data sets. Figures 2.b, 2.e, 3.b, and 3.e indicate the region sets, obtained through recursive splitting and edge detection. The resulting segmentations are given in Figures 2.c, 2.f, 3.c, and 3.f. For example, Figure 3.c shows three resulting segments (the specks appearing in the figures correspond to invalid data).

We conclude that the region merging probability provides a useful criterion both as a decision function for region merging, and as a termination criterion. For the decision to merge, the probability directly utilizes the information contained in the conditional densities of the image model, allowing the most confident decisions to be made first. Also, as a termination criterion, we can return a segmentation in which all merges were performed with probability greater than some value, P_c. One use of this is to perform a sequence of segmentations by setting P_c high, starting with a planar model, and increasing the degree of the model each time. We are presently applying the clustering techniques to an MRF texture model on intensity data.

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References

[1] A. K. Jain and R. C. Dubes. *Algorithms for Clustering Data.* Prentice Hall, Inc., Englewood Cliffs, NJ, 1988.

[2] J. Jolion, P. Meer, and S. Bataouche. Robust clustering with applications in computer vision. *IEEE Trans. Pattern Anal. Machine Intell.*, 13(8):791–801, August 1991.

[3] S. M. LaValle. A Bayesian framework for considering probability distributions of image segments and segmentations. Master's thesis, Univ. of Illinois, Urbana/Champaign, December 1992.

[4] S. M. LaValle and S. A. Hutchinson. Bayesian region merging probability for parametric image models. In *Proc. IEEE Conf. on Comp. Vision and Patt. Recog.*, 1993.

[5] J. F. Silverman and D. B. Cooper. Bayesian clustering for unsupervised estimation of surface and texture models. *IEEE Trans. Pattern Anal. Machine Intell.*, 10(4):482–496, July 1988.

[6] J. Zhang and J. W. Modestino. A model-fitting approach to cluster validation with applications to stochastic model-based image segmentation. *IEEE Trans. Pattern Anal. Machine Intell.*, 12(10):1009–1017, October 1990.